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Dissipative Diamagnetism—A Case Study for Equilibrium and Nonequilibrium Statistical Mechanics

Malay Bandyopadhyay[1] **and Sushanta Dattagupta**[1]

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Using the path integral approach to equilibrium statistical physics the effect of dissipation on Landau diamagnetism is calculated. The calculation clarifies the essential role of the boundary of the container in which the electrons move. Further, the derived result for diamagnetization also matches with the expression obtained from a time-dependent quantum Langevin equation in the asymptotic limit, provided a certain order is maintained in taking limits. This identification then unifies equilibrium and nonequilibrium statistical physics for a phenomenon like diamagnetism, which is inherently quantum and strongly dependent on boundary effects. In addition we have shown that our results are directly connected with fluctuation induced diamagnetic susceptibility of superconducting grains.

KEY WORDS: Landau diamagnetism, Dissipation, Mesoscopic system

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1. INTRODUCTION

Diamagnetism, which occurs as a result of the orbital motion of electric charges in the presence of a magnetic field, is an old and well studied problem. It was shown by Bohr and Van Leeuwen that when classical statistical mechanics are applied to the calculation of the diamagnetic moment, the answer is identically zero.(1) Thus, diamagnetism is an intrinsically quantum mechanical property, the treatment for which was provided by Landau after the advent of quantum mechanics.(2) There is an interesting issue of the role of the boundary within which the charges move,

[1] S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake City, Kolkata 700098, India; e-mail: malay@bose.res.in, sdgupta@bose.res.in

as was studied in depth by Van Vleck and Peierls.[3,4] While in classical statistical mechanics the contribution to the diamagnetic moment arising from the orbiting charges within the bulk of the container exactly cancels the contribution coming from the boundary-currents, this cancellation is incomplete in the quantum case, yielding a non-zero value of the diamagnetic moment. The boundary currents or edge currents are also important in the context of the quantum Hall effect.[5] In an earlier work,[6] referred here as I, we addressed the question of what happens to diamagnetism when there is dissipation present. Because the diamagnetic moment is proportional to the expectation value of the vector product of the operators $\mathbf{r}$ and $\mathbf{v}$, $\mathbf{r}$ being the position of the charge and $\mathbf{v}$ its velocity, the calculation was set up as a transport problem, much like the celebrated Drude conductivity of charge carriers.[7] Thus, the stationary form of the magnetic moment was obtained from the asymptotic (i.e. time $t \to \infty$) limit of the exact solution of an underlying quantum Langevin equation (QLE) for $\mathbf{r}$ and $\mathbf{v}$.[8] Naturally, the role of the boundary had to be carefully assessed by first solving the QLE in the presence of a confining boundary, then taking the $t \to \infty$ limit for the diamagnetic moment, and finally removing the boundary.

The QLE employed in I is in the spirit of the Caldeira Leggett model for which the harmonic oscillators are viewed to constitute a quantum bath that defines the temperature.[9] In this paper we present an alternative calculation of the diamagnetic moment, which is now viewed as a thermodynamic property, derivable from the derivative of the Gibbs partition function. Thus the full Hamiltonian comprising the charged particle in a magnetic field, the harmonic oscillators and their coupling, is treated in the canonical ensemble of equilibrium statistical mechanics. For reasons mentioned earlier, a confining boundary has to be also included, which is to be eliminated only after the derivative of the partition function is computed. In the present calculation, the temperature T is that of an 'invisible' bath in which the canonical system is viewed to be embedded. While in I we have considered only the ohmic dissipation in the Caldeira-Leggett model, both non-ohmic and ohmic cases are treated here. Although it is not surprising that the result derived here in the ohmic limit matches with the answer obtained in I, that was based on a time-dependent Brownian motion approach,[10] dissipative diamagnetism provides an elegantly pedagogical toy model within which equilibrium and nonequilibrium statistical mechanics can be holistically combined. We may point out that the combined effect of dissipation and confinement on Landau diamagnetism, the latter arising from coherent cyclotron motion of the electrons, is particularly relevant in the context of intrinsic decoherence in mesoscopic structures and fluctuation induced diamagnetic susceptibility and conductivity in superconducting structures[11–13] in view of heat bath induced influence.[5,14]

With the preceding background we organize the paper as follows. In Sec. 2 we discuss our model. Using the imaginary time path integral method, we reduce the infinite dimensional action to an effective two dimensional action by integrating out the environmental degrees of freedom. The equilibrium magnetization is derived in Sec. 3, wherein we also discuss the equivalence of our results with that of I and their relation with fluctuation induced diamagnetic susceptibility. In Sec. 4 we briefly describe the significance of our results and present some conclusions.

2. MODEL, FORMALISM AND EFFECTIVE ACTION

The starting point of I as indeed in this paper is the Feynman-Vernon[15] Hamiltonian for a charged particle e in a magnetic field $\mathbf{B}$:

$$\mathcal{H} = \frac{1}{2M}\omega_0^2\mathbf{x}^2 + \frac{1}{2M}\left(\mathbf{p} - \frac{e\mathbf{A}}{c}\right)^2 + \sum_{j=1}^{N}\left[\frac{1}{2m_j}\mathbf{p}_j^2 + \frac{1}{2}m_j\omega_j{}^2(\mathbf{x}_j - \mathbf{x}^2\right] , \quad (1)$$

where the first term is the Darwin[16] term representing a confining potential to recover the correct boundary contribution, $\mathbf{p}$ and $\mathbf{x}$ are the momentum and position operators of the particle, $\mathbf{p}_j$ and $\mathbf{x}_j$ are the corresponding variables for the bath particles, and $\mathbf{A}$ is the vector potential. We will work in the 'Symmetric Gauge.' The bilinear coupling between $\mathbf{x}$ and $\mathbf{x}_j$ as envisaged in Eq. (1) has been the hall mark of the Caldeira-Leggett approach to dissipative quantum mechanics.[9,17] Further, it has been shown by Chang and Chakravarty that a fermionic heat bath for electron-hole excitations near the Fermi surface, as appropriate for a metal, can indeed be represented by bosonic operators, which are just the second quantized forms of the harmonic oscillator variables of the Caldeira Leggett model, especially when Ohmic dissipation is assumed.[18] Assuming the $\mathbf{B}$ field to be along the z-axis, all the vectors in Eq. (1) can be taken to lie in the xy-plane. Thus, the vector $\mathbf{x}$ has two components x and y etc.

Using the imaginary time path integral method we calculate the effective Euclidean action. The partition function of the whole system is given by

$$\mathcal{Z} = \int D[\mathbf{x}]\exp\left[-\frac{\mathcal{A}_e[\mathbf{x}]}{\hbar}\right] , \quad (2)$$

where $\mathcal{A}_e[\mathbf{x}]$ is the effective Euclidean action and the functional integral is over all periodic paths with period $\hbar\beta$. The free energy is then given by

$$\mathcal{F} = -\frac{1}{\beta}\ln\mathcal{Z} . \quad (3)$$

The important thermodynamic quantity, viz., the magnetization can easily be obtained by taking the first derivative of $\mathcal{F}$ with respect to the magnetic field B, applied along the z-axis:

$$\mathcal{M}_z = -\frac{\partial\mathcal{F}}{\partial B} . \quad (4)$$

Having laid down the background to the calculation of diamagnetism we pose and answer the following question in this paper. Should we not be able to calculate the equilibrium magnetization directly from Eq. (1) by following the usual Gibbsian statistical mechanics in which all the terms in Eq. (1) are treated on the same footing and there is no separation between what is a system and what is a bath? If the answer to this question is in the affirmative and the resultant magnetization matches with the result derived in I in the 'equilibrium limit' that would indeed make the Brownian motion approach of I equivalent to the usual statistical mechanics method.

Our method of calculation is based on the functional integral approach to statistical mechanics which we find to be the most convenient tool for studying charged particle dynamics in a magnetic field.[19–23] The canonical operator $\exp(-\beta\mathcal{H})$ is related to the time evolution operator $\exp(-\frac{i\mathcal{H}t}{\hbar})$ by an analytic continuation procedure known as Wick's rotation $t = -i\hbar\beta$. So in order to obtain the Euclidean action we have to analytically continue to imaginary time $\tau = it$. The Euclidean action corresponding to the Hamiltonian in Eq. (1) can be written as:

$$\mathcal{A}_e = \int_0^{\hbar\beta} d\tau[\mathcal{L}_S(\tau) + \mathcal{L}_B(\tau) + \mathcal{L}_I(\tau)], \tag{5}$$

where the subscripts S, B and I stand for 'system,' 'bath' and 'interaction' respectively. The corresponding Lagrangians are enumerated as:

$$\mathcal{L}_S(\tau) = \frac{M}{2}\left[\dot{\mathbf{x}}^2(\tau) + \omega_0^2\mathbf{x}^2(\tau) - i\omega_c(\mathbf{x}(\tau)\times\dot{\mathbf{x}}(\tau))_z\right], \tag{6}$$

where $\omega_c = \frac{eB}{Mc}$, is the cyclotron frequency,

$$\mathcal{L}_B(\tau) = \sum_{j=1}^{N}\frac{1}{2}m_j\left[\dot{\mathbf{x}}_j^2(\tau) + \omega_j^2\mathbf{x}_j^2(\tau)\right], \tag{7}$$

$$\mathcal{L}_I(\tau) = \sum_{j=1}^{N}\frac{1}{2}m_j\omega_j^2[\mathbf{x}^2(\tau) - 2\mathbf{x}_j(\tau)\cdot\mathbf{x}(\tau)]. \tag{8}$$

Since the path $x(\tau)$ has imaginary time periodicity $x(\hbar\beta) = x(0)$, we can perform imaginary time Fourier series expansion of system variables and bath variables as follows:

$$\mathbf{x}(\tau) = \sum_n \tilde{\mathbf{x}}(\nu_n)e^{-i\nu_n\tau}, \tag{9}$$

$$\vec{\mathbf{x}}_j(\tau) = \sum_n \tilde{\mathbf{x}}_j(\nu_n)e^{-i\nu_n\tau}, \tag{10}$$

where the Bosonic Matsubara frequencies ν_n are given by

$$\nu_n = \frac{2\pi n}{\hbar\beta}, \quad n = 0, \pm 1, \pm 2, \ldots. \tag{11}$$

Using Eqs. (9) and (10) and following the detailed treatment given by Weiss[22] the system-part of the action in terms of Fourier components is:

$$\mathcal{A}_e^s = \frac{M}{2}\hbar\beta \sum_n \left[\left(\nu_n^2 + \omega_0^2\right)(\tilde{\mathbf{x}}(\nu_n) \cdot \tilde{\mathbf{x}}^*(\nu_n)) + \omega_c \nu_n (\tilde{\mathbf{x}}(\nu_n) \times \tilde{\mathbf{x}}^*(\nu_n))_z\right]. \tag{12}$$

Further the combined contributions of the bath and the interaction terms to the action can be written as:

$$\mathcal{A}_e^{B-I} = \frac{M}{2}\hbar\beta \sum_n \xi(\nu_n)(\tilde{\mathbf{x}}(\nu_n) \cdot \tilde{\mathbf{x}}^*(\nu_n)), \tag{13}$$

where

$$\xi(\nu_n) = \frac{1}{M}\sum_{j=1}^{N} m_j \omega_j^2 \frac{\nu_n^2}{\left(\nu_n^2 + \omega_j^2\right)}. \tag{14}$$

Introducing the spectral density for bath excitations as:

$$J(\omega) = \frac{\pi}{2}\sum_{j=1}^{N} m_j \omega_j^3 \delta(\omega - \omega_j), \tag{15}$$

we may rewrite

$$\xi(\nu_n) = \frac{2}{m\pi}\int_0^\infty d\omega \frac{J(\omega)}{\omega} \frac{\nu_n^2}{\left(\nu_n^2 + \omega^2\right)}. \tag{16}$$

Now, combining Eq. (13) with Eq. (12), the full action can be expressed as:

$$\mathcal{A}_e = \frac{M}{2}\hbar\beta \sum_n \left[\left(\nu_n^2 + \omega_0^2 + \nu_n \tilde{\gamma}(\nu_n)\right)(\tilde{\mathbf{x}}(\nu_n) \cdot \tilde{\mathbf{x}}^*(\nu_n)) + \omega_c \nu_n (\tilde{\mathbf{x}}(\nu_n \times \tilde{\mathbf{x}}^*(\nu_n))_z\right], \tag{17}$$

where the 'memory-friction' is given by

$$\tilde{\gamma}(\nu_n) = \frac{2}{M\pi}\int_0^\infty d\omega \frac{J(\omega)}{\omega} \frac{\nu_n}{\left(\nu_n^2 + \omega^2\right)}. \tag{18}$$

Note that $\tilde{\mathbf{x}}(\nu_n)$ is a two-dimensional vector $(\tilde{x}(\nu_n), \tilde{y}(\nu_n))$. Introducing then normal modes:

$$\tilde{z}_+(\nu_n) = \frac{1}{\sqrt{2}}(\tilde{x}(\nu_n) + i\tilde{y}(\nu_n))$$

$$\tilde{z}_+(\nu_n) = \frac{1}{\sqrt{2}}(\tilde{x}(\nu_n) - i\tilde{y}(\nu_n)), \tag{19}$$

Equation (17) can be rewritten in a 'separable' form:

$$\mathcal{A}_e = \frac{M}{2}\hbar\beta\sum_n \big[\big(\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n) + i\omega_c\nu_n\big)(\tilde{z}_+(\nu_n)\tilde{z}_+^*(\nu_n))$$
$$+\big(\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n) - i\omega_c\nu_n\big)(\tilde{z}_-(\nu_n)\tilde{z}_-^*(\nu_n))\big]. \tag{20}$$

Equation (20) is the required effective Euclidean action.

3. FREE ENERGY AND MAGNETIZATION

In this section we employ the action given by Eq. (20) to first calculate the canonical partition function and from it, the thermodynamic free energy. In doing this calculation we tacitly assume *a la* Gibbs that the entire Hamiltonian, described by Eq. (1), is embedded in a thermal bath that defines the temperature of the system. This is in contrast to the QLE approach in I which assumes that it is the subsystem alone, comprising the electron in a magnetic field, that is immersed in a heat bath of quantum harmonic oscillators.

From Eq. (20) the partition function can be written as:

$$\mathcal{Z} = \frac{2\pi}{M\beta}\prod\nolimits_n \big[\big(\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n)\big)^2 + \omega_c^2\nu_n^2\big]^{-1}, \tag{21}$$

where, we have used the definition of partition function as given in Eq. (2). In view of Eq (3) the Helmholtz Free energy $\mathcal{F}$ can be deduced from Eq. (21) as

$$\mathcal{F} = \frac{1}{\beta}\,\mathrm{In}\left(\frac{M\beta\omega_0^4}{2\pi}\right) + \frac{2}{\beta}\sum_{n=1}^{\infty}\mathrm{In}\big[\big(\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n)\big)^2 + \omega_c^2\nu_n^2\big], \tag{22}$$

where the first term is independent of the magnetic field and owes its existence purely to the Darwinian constraining potential. Equation (22) contains all the thermodynamic properties, the most important of which is the *magnetization* given by the negative derivative of $\mathcal{F}$ with respect to B:

$$\mathcal{M}_z = -\sum_{n=1}^{\infty}\frac{\frac{4}{\beta B}\omega_c^2\nu_n^2}{\big[\big(\nu_n^2 + \omega_0^2 + \nu + n\tilde{\gamma}(\nu_n)\big)^2 + \omega_c^2\nu_n^2\big]}, \tag{23}$$

thus yielding a manifestly negative magnetization, the hallmark of diamagnetism. We conclude that the dissipative system of a charged quantum oscillator in an external magnetic field is still diamagnetic. Equation (23) identically matches with the asymptotic ($t \to \infty$) limit of the expression obtained by Li *et al.*[8] from a quantum Langevin equation formulation. Equation (23) can be recast in terms of dimensionless parameters like $\zeta(=\frac{\hbar\tilde{\gamma}(\nu_n)}{2k_BT})$, $\nu_c(=\frac{\hbar\omega_c}{2k_BT})$ and $\nu_0(=\frac{\hbar\omega_0}{2k_BT})$ as follows:

$$\mathcal{M}_z = -\frac{B}{k_BT}\left(\frac{e\hbar}{Mc}\right)^2\sum_{n=1}^{\infty}\frac{1}{\big[n\pi + \frac{\nu_0^2}{n\pi} + \zeta\big]^2 + \nu_c^2} \tag{24}$$

The Eq. (24) of the present manuscript also yields the asymptotic result of I, for $\nu_0 = 0$ (cf. Eq. (19) of I). We demonstrate this below, for ohmic dissipation, since only the latter case was considered in Ref. I. In the so-called ohmic dissipation model[9]

$$J(\omega) = M\gamma\omega. \tag{25}$$

Now Eq. (19) of I can be recast as follows:

$$\mathcal{M}_z = \frac{|e|\hbar}{2Mc}\left\{\sum_{n=1}^{\infty}\frac{4n\pi\zeta\nu_c}{\left(\nu_c^2+\zeta^2-n^2\pi^2\right)^2+4n^2\pi\nu_c^2}\right. \left. +\Re\left[\frac{1}{(\nu_c-i\zeta)}-\coth(\nu_c-i\zeta)\right]\right\}. \tag{26}$$

Note that the term inside the square parentheses is just the Landau contribution with however a *complex* cyclotron frequency with damping ζ as the imaginary component. Over and above this is the further contribution, solely dependent on damping, given by the first term (involving a summation over n) within the curly brackets. Using the identity

$$\coth(z) = \frac{1}{z} + \sum_{n=1}^{\infty}\frac{2z}{(z^2+n^2\pi^2)}, \tag{27}$$

one can rewrite Eq. (26) as follows:

$$\mathcal{M}_z = \frac{|e|\hbar}{2Mc}\left\{\sum_{n=1}^{\infty}\frac{4n\pi\zeta\nu_c}{\left(\nu_c^2+\zeta^2-n^2\pi^2\right)^2+4n^2\pi\nu_c^2}\right. \left. -\Re\sum_{n=1}^{\infty}\frac{2(\nu_c-i\zeta)}{(\nu_c-i\zeta)^2+n^2\pi^2)}\right\}. \tag{28}$$

After some algebra one can express $\mathcal{M}_z$ as

$$\mathcal{M}_z = -\frac{|e|\hbar}{Mc}\nu_c\sum_{n=1}^{\infty}\frac{1}{\nu_c^2+(\zeta+n\pi)^2}, \tag{29}$$

which equals Eq. (24) in the limit $\nu_0 \to 0$.

It has often been felt, starting from the old Larmor theory of diamagnetism,[3] that $\mathcal{M}_z$ ought to be simply proportional to the mean-squared electron radius, as it connects to the square of the vector potential **A** occuring in the Hamiltonian in Eq. (1).[12] In order to explore this connection, we now switchover to the calculation of the dispersion of position in equilibrium states which is given by

$$\langle x^2 \rangle = \frac{1}{M\beta\omega_0^2} + \frac{2}{M\beta}\sum_{n=1}^{\infty}\frac{\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n)}{\left[(\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n))^2 + (\nu_n\omega)^2\right]}. \tag{30}$$

From Eq. (30) it is evident that the mean square radius $\langle x^2 \rangle$ decreases monotonically with the increasing strength of the dissipative factor ($\tilde{\gamma}(\nu_n)$). Combining Eq. (30) with Eq. (23) we obtain

$$\mathcal{M}_z = -\frac{2B}{Mc}\left[\langle x^2 \rangle - \frac{1}{M\beta\omega_0^2} - \frac{2}{M\beta}\sum_{n=1}^{\infty}\frac{\omega_0^2 + \nu_n\tilde{\gamma}(n_n)}{\left[\nu_n^2 + \omega_0^2 + \nu_n\tilde{\gamma}(\nu_n))^2 + (\nu_n\omega_c)^2\right]}\right]. \tag{31}$$

Even after ignoring the classical equipartition term (i.e. the first term on the right of Eq. (30)) we find that $\mathcal{M}_z$ in magnitude is further decreased from $\langle x^2 \rangle$ by a nontrivial damping dependent term given by a summation over n. This implies actually an increase beyond the value of $\langle x^2 \rangle$, in view of the overall positive sign in front of the sum over n. The origin of this additional contribution may be traced to the fact that the treatment provided above is an exact one, including the linear term in **A**. Be that as it may, the decrease in magnitude of diamagnetization, as the damping increases, may be interpreted to be due to the squeezing of $\langle x^2 \rangle$ due to dissipation. Thus the present effect is related to how dissipation diminishes the fluctuation induced diamagnetic susceptibility (above T_c) of superconducting grains.

One can also re-express the equilibrium dispersion Eq. (30) in terms of dimensionless parameters ζ, ν_c and ν_0

$$\langle x^2 \rangle = \frac{\hbar^2}{4Mk_BT}\left[\frac{1}{\nu_0^2} + \sum_{n=1}^{\infty}\frac{1 + \left(\frac{\nu_0}{n\pi}\right)^2 + \frac{\zeta}{n\pi}}{\left[n\pi + \frac{\nu_0^2}{n\pi} + \zeta\right]^2 + \nu_c^2}\right] \tag{32}$$

Now to make our theoretical analysis more accessible and more interesting we numerically plot our main results i.e Eq. (24) and Eq. (32). We consider both the frequency dependent and independent damping cases i.e both nonohmic (J(ω)$\sim\omega^3$) and ohmic (J(ω)$\sim\omega$) dissipation. We plot in Fig. 1 magnetization $\mathcal{M}_z$ versus dimensionless damping parameter ζ for different values of ν_c in accordance with Eq. (24). It is seen that $\mathcal{M}_z$ monotonically approaches zero for large value of ζ although this approach is slower the larger ν_c is. A large value of ν_c gives strong quantum effect which ultimately gives classical like effects when dissipation ζ is strong. In Fig. 2 we plot equilibrium position dispersion versus ζ for different values of ν_c. Here $\langle x^2 \rangle$ decreases monotonically to zero for large value of ζ although the behavior of $\langle x^2 \rangle$ is different from $\mathcal{M}_z$.

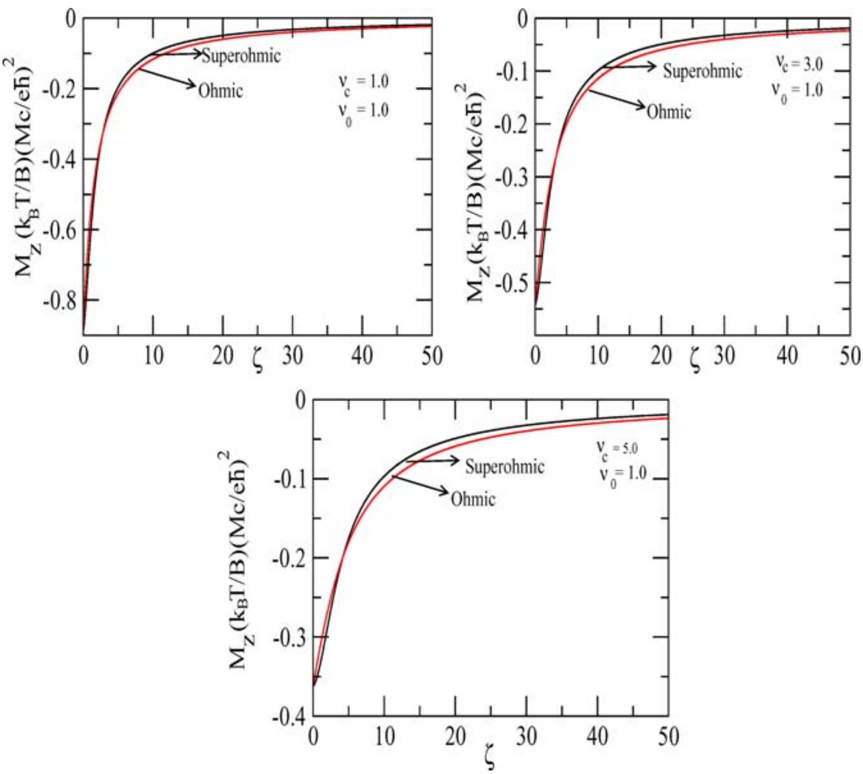

Fig. 1. Plot of $\frac{2k_B T}{B}(\frac{Mc}{e\hbar})^2\mathcal{M}_z$ versus the damping parameter ζ for both ohmic (J(ω)$\sim\omega$) and nonohmic (J(ω)$\sim\omega^3$) cases.

4. SUMMARY AND CONCLUSION

Equation (23) embodies several intriguing results which deserve special comments: (1) Diamagnetic susceptibility in small particles is proportional to the mean squared radius $\langle x^2 \rangle$ of the charged particles in the grain; $\langle x^2 \rangle$ is squeezed due to dissipation and hence fluctuation induced diamagnetic susceptibility of superconducting grains also decreases. This is an important message of the present work. (2) It has often been seen that although the approach to equilibrium does depend on relaxation parameters such as damping, the equilibrium results themselves are independent of such parameters.(24,25) The diamagnetization is one of the rare equilibrium properties which depends directly on the damping parameter γ that characterizes the dissipative dynamics of the underlying Hamiltonian. The reason is, like in the much studied problem of quantum dissipation of a harmonic oscillator,(26) the system-bath coupling is so strong that it needs an exact

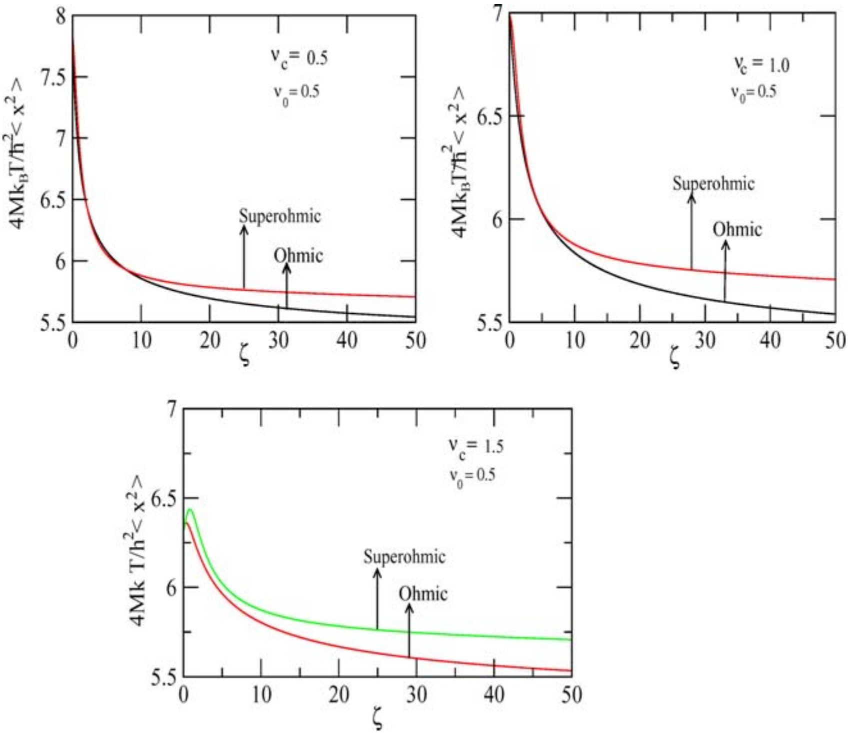

Fig. 2. Plot of equilibrium position dispersion $\langle x^2 \rangle$ in unit of $\frac{\hbar^2}{4Mk_BT}$ for both ohmic (J(ω)$\sim\omega$) and nonohmic (J(ω)$\sim\omega^3$) dissipation cases.

treatment. Thus the degrees of freedom of the entire many body system are inexorably entangled with each other and therefore, it is no longer meaningful to separate what is a system from what is a bath. In this context we should mention that the derivation of Boltzmann distribution $\exp(-\beta\mathcal{H})$ only works in the limit of vanishing interaction strength[27] and indeed this has been discussed for the Caldeira-Leggett model by Benguria *et al.*[28] But ours is a calculation in which the system-bath interaction has been treated exactly. (3) Diamagnetism as a material property is seen to have components of thermodynamics and transport phenomena. The thermodynamic nature of the property is rooted on its being able to be calculated from the free energy, as shown here. On the other hand, diamagnetism, like the Drude conductivity,[7] is also based on transport mechanism in that it is related to the expectation value of the operator ($\mathbf{r} \times \mathbf{v}$) (see I). (4) Normally, in statistical mechanics, a thermodynamic limit is taken as a result of which surface contributions to bulk become irrelevant. However, for diamagnetism the surface enters crucially, as argued above; even though, there are fewer surface electrons than in the bulk, their contribution to the operator $\mathbf{r}$ in $(\mathbf{r} \times \mathbf{v})$ is substantial. A remarkable feature of diamagnetism is the need to first calculate the magnetization in the thermodynamic limit and then switch the boundary off i.e. by setting $\omega = 0$. Because for a mesoscopic system surface effects are non-negligible, the present study has a bearing on our understanding of mesoscopic structures. (5) Finally, it has been argued by Jayannavar and Kumar,[29] not only is there no classical diamagnetism—due to the Bohr-Van Leeuwen theorem—there is no dissipative classical diamagnetism either. Thus, the time-dependent, classical diamagnetization relaxes to zero, a damping-independent result. Therefore, we emphasize once again that the appearance of damping terms in equilibrium answers, as discussed under points (2) and (5), is an intrinsically quantum aspect.

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